BDS-test as a benchmark for noise cleaning

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Methods of noise cleaning have to pass statistical tests which should ensure, that they do not eliminate relevant parts of the signal. We propose the BDS-test - originally used to distinguish between stochastic and deterministic data - to measure how strong noise cleaning affects the deterministic nature of data. Possibilities and limitations of the BDS-test for different data-types are presented.

1 Introduction

Measurement noise refers to the corruption of observations by errors that are independent of the dynamics of the system. We [1] recently proposed a noise cleaning algorithm based on a work of Grassberger [2]. However, methods of noise cleaning have to be tested to show that noise-removal did not affect the signal. We propose to use the BDS-test – named after Brock, Dechert and Scheinkman and developped as a decision tool whether a given signal is deterministic or stochastic [3] – for this task.

2 Methods

Let x_t be a time series of measurements of length T. The coordinate delay method [4] builds vectors out of the time series, which form a manifold in the embedding space. Noise has a "bluring" effect on this manifold and noise cleaning can be interpretet as its "resharpening". If the total number of embedded vectors is N, the quantity $C_N^{(m)}(\epsilon) = \frac{1}{N^2}\sum_{i\neq j}\theta(\epsilon - \|\xi_i - \xi_j\|)$ is called correlation integral [5], $\theta(x)$ is the heavyside-function with $\theta(x) = 0$ for $x < 0$ and $\theta(x) = 1$ for $x \geq 0$. Based on the correlation integral, we introduce a simplified, non-normalised BDS test statistics, that is sufficient for practical purposes:

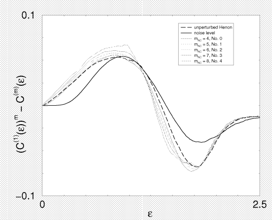
$$\tilde{W}^{(m)}(N,\epsilon) = C^{(m)}(N,\epsilon) - C^{(1)}(N,\epsilon)^m \qquad (1)$$

Under the null hypothesis that $\{x_t\}$ are independent and identically distributed, $\tilde{W}^{(m)}(N,\epsilon)$ converges towards the the normal distribution with zero mean. When the empirical mean of $\tilde{W}^{(m)}(N,\epsilon)$ significantly deviates from zero, the underlying dynamics of the system is deterministic.

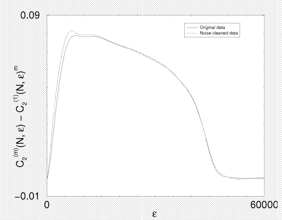
The idea of applying the BDS-test to check the quality of noise cleaning is as follow: Our noise cleaning algorithm using a projection dimension m_0 (the dimension of the subspace to which the vectors are projected) is applied to an embedded (the embedding dimension is m_{NC}) noisy time series. From the noise-cleaned vectors a new time series is generated which is again embedded in an embedding space of dimension m_{BDS}. After determining appropriate values for m_{BDS}, m_{NC} is varied for a fixed m_0. If the curves converge in a way, that the distances $C^{m_{BDS}}(N,\epsilon) - [C^{1_{BDS}}(N,\epsilon)]^{m_{BDS}}$ for those m_{BDS} where the curves converge is much smaller than the distance between these curves and the curve of the uncleaned time series, the noise cleaning did not significantly change the signal.

3 Results

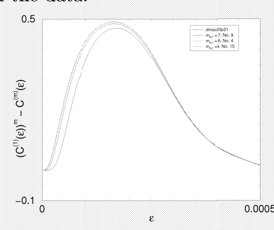
1. The Henon model system: First we use a mathematical test system to show the power of the test in a well-defined context. For the classical parameter values $\alpha = 1.4$ and $\beta = 0.3$ a strange attractor is obtained from the Henon map. Time series generated by this system were contaminated by additive gaussian noise. Then our noise cleaning algorithm and finally the BDS-test have been applied. Figure 1 shows the result for $m_0 = 2$, $m_{BDS} = 6$ and m_{NC} varying from 4 to 8 after 6 iterations of noise cleaning. The BDS-test confirms that the deterministic structure has only slightly been influenced.



2. NEURON modeling data: Our noise cleaning algorithm has been used for data of a model of a recurrent neocortical network [6]. As a reference, a purely stochastic Gaussian that has identical moments has been considered. For the network-data, the response is substantially different, indicating that the observed irregularities in the spiking behaviour are notstochastic, but are of deterministic, low-dimensional nature. Moreover, the difference exhibited by the noise-cleaned BDS-distribution functions from the original data is minimal. This implies that noise cleaning did only marginally change the statistical behaviour of the original files.



3. Data from the auditory system of insects: Robert et al. [7] explore the mechanisms of active amplification in the auditory systems of the mosquitoe *T. brevipalpis.* We analysed the data of antenna vibrations with our noise cleaning algorithm. First, a closer look showed problems due to the discretisation of the data as a result of the resolution of the measurement apparatus. Noise cleaning removed this discretisation what led to the suspect, that relevant aspects of the data might be removed. Nevertheless, by applying the BDS-test to the noise cleaned data, we see a convergence of the curves for dimensions around 8, which shows that noise cleaning did not affect the deterministic nature of the data.



4 Discussion

Noise is a relevant problem in time series of neurobiological data like interspike intervals. However, noise cleaning can only successfully applied, when the data is not affected in a way, that wrong conclusion are drawn. Our novel application of the BDS-test is able to ensure the quality of noise cleaning in different cases.

5 Literature

[1] Kern A., Steeb W.-H., Stoop R. Projective Noise Cleaning with Dynamic Neighborhood Selection. Int. J. Mod. Phys. C 11(1), 2000: 125-146.
[2] Grassberger P., Hegger R., Kantz H., Schaffrath C., and Schreiber T. On noise reduction methods for chaotic data, Chaos 3, 1993: 127-141.
[3] Brock W.A., Dechert W.D. Theorems on Distinguishing Deterministic and Random Systems. Depart. of Econ. Univ. of Wisconsin, Madison and Depart. of Econ. Univ. of Houston, Houston, Texas, 1985
[4] Kantz H., Schreiber T. Nonlinear Time Series Analysis. second ed. Cambridge University Press, 2000.
[5] Grassberger P., Procaccia I. Measuring the strangeness of strange attractors. Phys. 9D, 1983: 189-208.
[6] Blank D.A. Firing rate amplification and collective bursting in models of recurrent neocortical networks. PhD thesis, Swiss Federal Institute of Technology, 2001.
[7] Gopfert, M., Robert D. Active auditory mechanics in mosquitoes. Proc. R. Soc. Lond. B 268, 2001: 333-339.

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