A SPIN-BASED MEASURE OF THE COHERENCE OF BELIEF SYSTEMS

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Abstract— *In social science and humanities, coherence is an important but vaguely defined concept used for understanding the structure of belief systems and their physical representations like political parties. In our contribution, we provide a specified definition and measure of coherence based on superparamagnetic clustering. The measure captures the intuition, that coherence includes both a static component (diversity of sub-groups in the system) and a dynamic component (stability of the system under stress). We apply our measure of coherence to data representing the political beliefs of candidates in the Swiss national elections of 2003 and 2007 and explain the split-up of the Green party in 2004 as a result of lacking coherence both in its static and dynamic component.*

I. Introduction

The term 'coherence' has different interpretations in scientific disciplines. Rigid definitions adapted to specific problems are found in quantum physics and signal processing. But the term is also used in social sciences and humanities, where it describes the logical and/or semantical coherence of the propositions forming a belief system [1]. However, the concept of coherence in these settings is mostly vaguely defined. This lack of precision is dissatisfactory, because social groupings like political parties are accompanied with the formation of belief systems representing the shared opinions of party members on important issues of social organization [2]. Therefore, the degree of coherence of the opinions between the members of a party is a component that may explain changes within the party up to a culmination into its split.

This contribution outlines a quantitative notion of coherence in three steps. First, we outline the main intuitions that a measure of coherence should capture. Second, we define our measure of coherence based on superparamagnetic clustering and introduce its properties using toy data. Third, we apply our measure to data representing the political beliefs of candidates in the Swiss national elections of 2003 and 2007.

II. The intuition of coherence

A belief system is a system of meaningful declarative sentences regarding states of the world (propositions), which a individual of a group holds to be true. In the following, we constrain ourselves to political beliefs referring to states of societal organization (e.g.: "Families pay too much taxes."). These beliefs are represented by persons. Belief systems that form the ideological basis of a political party are considered to have the property of coherence – i.e. the party members share similar beliefs regarding propositions that refer to the political organization of society at least to some degree [3]. Factual incoherence is taken as a possible explanation for tensions in political parties that may culminate in a split of the party. What do we mean when we say that a belief system is coherent?

To analyze coherence, usually 'important beliefs' within a system are identified by qualitative analysis and their mutual logical consistency or semantic affinity is checked. This method is not satisfactory, as the selection process tends to be arbitrary and the analysis delivers only a snapshot of the system's structure. It disregards the fact, that political parties and their underlying belief systems are objects of changing con-

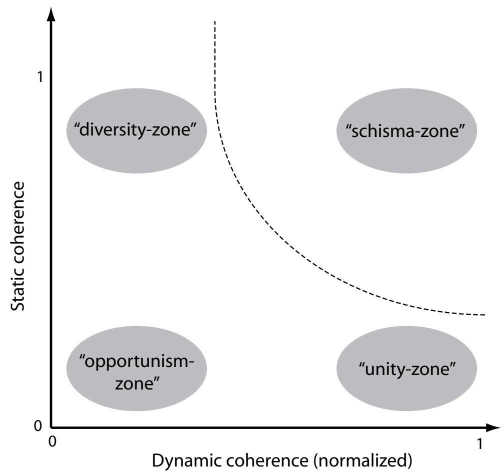

Fig. 1. Conceptual outline of a coherence measure along two dimensions.

ditions within society. Coherence has also a dynamic component, reflected for example by the number of persons that (virtually) 'drop out' when pressure increases. Furthermore, the static component should take into account the whole diversity of 'sub groups' of beliefs (represented by party members). A party understood as a physical representation of a belief system is therefore coherent along two dimensions:
1. *Dynamic component:* The stability of the party under pressure.
2. *Static component:* The diversity of sub groups within the party.

Coherence as being composed of two numeric values allows to identify several distinguished states (see Fig. 1): Static and dynamic coherence may be low. Such a party has a low inner structure and its members tend to leave the party when pressure is high ('opportunists'). Low static and high dynamic coherence may indicate a party with high degree of unity. If static coherence is high and dynamic coherence is low, the party has a high diversity but lacks a strong and stable core. When static and dynamic coherence are high, the party's belief system has a structure that makes the party vulnerable to a split, as several strong sub-groups exist which are inherent stable ('schismazone'). We will now operationalize this intuition using the framework of superparamagnetic clustering.

III. THEORY: SPIN-BASED COHERENCE MEASURE

A. *Superparamagnetic clustering*

The superparamagnetic clustering algorithm [4] SPC is inspired by a self-organization phenomenon in magnetic spin systems: in an inhomogeneous spin system, clusters of correlated spins can emerge, corresponding to groups of spins with strong couplings. Upon an increase in temperature, i.e., an increase in pressure on the system, these clusters decay into smaller units in a cascade of (pseudo-)phase transitions. For data clustering, we map a data set onto a spin system. Each data item is represented by a Potts spin variable s_i with possible values in $\{1, ..., q = 10\}$. Each spin is coupled to its $k = 5$ nearest neighbors, where for given distances $d_{ij} = d_{ji}$ between spins the couplings are determined according to

$$J_{ij} = J_{ji} = \frac{1}{k} \exp\left(\frac{-d_{ij}^2}{2a^2}\right). \tag{1}$$

a is the average distance between neighbors. Each spin configuration s is associated with the probability

$$p(s) = \frac{1}{Z(T)} \exp\left(\frac{-H(s)}{T}\right) \tag{2}$$

with the Hamiltonian $H(s) = \sum_{(ij)} J_{ij}(1 - \delta_{s_i s_j})$ and the normalization constant $Z(T)$. The parameter T represents the system temperature. At a given T, clusters are detected by means of the pair correlation $G_{ij} = \sum_s p(s)\delta_{s_i s_j}$, approximately calculated by a Monte Carlo procedure [4]. If $G_{ij} > \Theta = 0.7$, then s_i and s_j belong to the same cluster.

Natural clusters, i.e., clusters with strong homogeneous couplings, become manifest in their stability over a substantial range of T. Hence the T-stability provides a natural measure of cluster 'coherence'. This fact is exploited by the sequential superparamagnetic clustering algorithm SSC [5]. In this approach, the most stable cluster is extracted and it, as well as the residual set, is reclustered. The procedure is repeated, providing a natural dendrogram with a cluster hierarchy. Details can be found in [5].

The couplings between spins and hence the clustering results critically depend on the distances d_{ij} between the data points. The choice of the distance function is guided by the problem type one wants to solve and usually relies on the methodology of the scientific discipline in which one operates. It needs not necessarily fulfil all axioms of a mathematical distance.

B. *Defining coherence*

Using the framework of SPC and SSC for defining the coherence of belief systems requires in a fist step a definition of the data points and their mutual distance. For n data points, the application of the distance measure leads to a $n \times n$ distance matrix that serves as input for the clustering algorithm.

The dynamic component of coherence $C_{dynamic}$ is calculated in the SPC framework. It is evaluated as the disintegration of the largest cluster for increasing T (this involves the assumption that the largest cluster represents the 'core' of the belief system that disintegrates under stress). Let $CS(t)$ be the size of the largest cluster for $T = t$. We assume $CS(0) = n$ and then calculate $CS(t)$ stepwise for $t = \Delta T, 2\Delta T, \ldots$ until after the l-th step we have $CS(l\Delta T) = 1$, i.e. the large cluster has vanished completely. In this way we calculate a step-function approximation of the decay curve, whose integral serves as a measure for dynamic coherence. To make the integral comparable, it is normalized with n and l, leading to the definition

$$C_{dynamic} = \sum_{i=0}^{l-1} \frac{CS(i\Delta T) + CS((i+1)\Delta T)}{2nl} \tag{3}$$

$C_{dynamic}$ is close to 1 for a cluster that remains intact for a long time and then disintegrates rapidly for

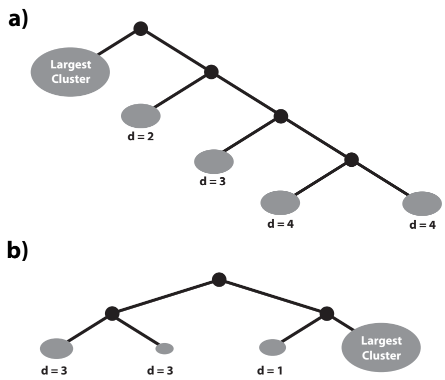

Fig. 2. Illustration of the network distance. a) refers to the example 'blue' and b) to the example 'green' (toy data, see below), the ovals are scaled with cluster size.

high T, whereas $C_{dynamic}$ is close to 0 for a cluster that disintegrates rapidly in which only a small core is stable for a longer time.

The static component of coherence C_{static} is calculated using SSC that results in a dendrogram in which the size of each of the k sub-cluster is evaluated. We consider the largest cluster $\bar{c}$ as the 'core' of the system. $C_{dynamic}$ is calculated as the sum of the distance of each cluster c_i from the largest cluster in the network weighted with its size $|c_i|$. The 'network distance' d_i is the number of bifurcation points between $\bar{c}$ and c_i (see Fig. 2). Both the maximal network distance d_{max} as well as the size of the largest cluster serve as normalization factors, leading to the definition

$$C_{static} = \sum_{i=1}^{k} \frac{\mathsf{d}_i}{\mathsf{d}_{max}} \cdot \frac{|c_i|}{|\bar{c}|} \tag{4}$$

Remind that C_{static} is not normalized to 1. Its value is 0, if SSC does not reveal any sub-cluster and it is close to 0 if only small clusters emerge. However, many large clusters that have a large network distance from the largest cluster increase C_{static}. We will now calculate both the static and the dynamic component of coherence for toy data.

C. Coherence in toy systems

To exemplify the measure, we will use toy data where beliefs are represented as points in a 2D space and the distance is Euklidean (the colors refer to the display of the results in Fig. 3):

- Red: A homogeneous 2D lattice point cluster: As expected, this cluster remains stable for a large T interval and then collapses fast. Hence $C_{dynamic}$ is close to 1 and C_{static} is 0, as no sub-clusters are present.
- Orange: A large homogeneous 2D lattice point cluster with a small homogeneous 2D lattice point cluster close to it. As expected, the small cluster splits of quite soon for increasing T whereas the large, remaining cluster remains stable for a large T interval until it collapses. Hence $C_{dynamic}$ is still close to 1 and C_{static} is close to 0, as only one sub-cluster is present.
- Yellow: Two 2D lattice point clusters of the same size: As expected, they split pretty soon and each of them remains stable for a large T interval (only one of them is tracked). Hence, $C_{dynamic}$ is close to 0.5 and C_{static} is large, as there are two large clusters present.
- Green: A small 2D lattice point cluster embedded in noise (points with randomly chosen x and y values). The noise drops off fast, whereas the small cluster remains stable for a large T interval. Hence, $C_{dynamic}$ is close to 0 and C_{static} is small as well, as most noisy points find themselves in a large (unstable) cluster.
- Blue: A large 2D lattice point cluster with several smaller 2D lattice point cluster in its neighborhood with decreasing distance that drop off sequentially. Both $C_{dynamic}$ and C_{static} are rather large.
- Light blue: A Gaussian cluster (points with normal distributed coordinates), where the points drop off continuously for increasing T. $C_{dynamic}$ is close to 0.5 and C_{static} is 0, as there are no sub-clusters.

Remind that two examples lie in the 'schisma-zone': the example 'yellow' with two clusters of equal size and the example 'blue' where half of the points are again organized in several stable sub-clusters.

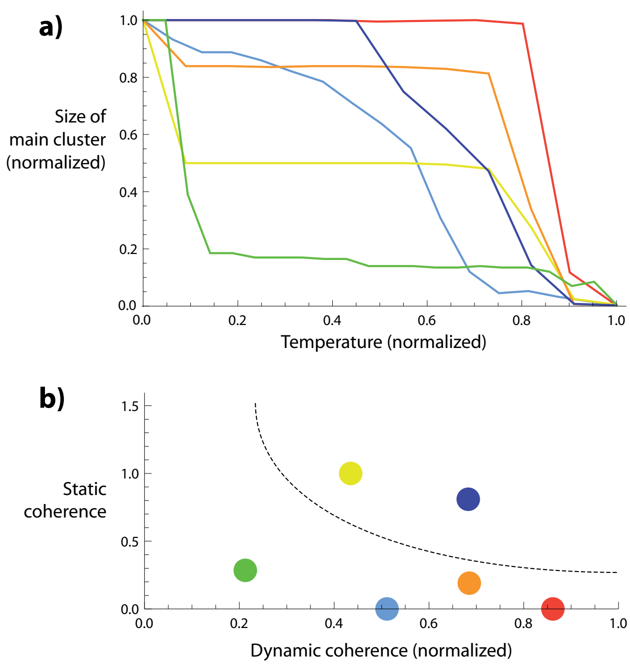

Fig. 3. Decay diagram for calculating the dynamic coherence (a) and static and dynamic coherence for toy data.

IV. Application: Coherence of parties

A. Data description

Our data originate from the Smartvote project [6]. As part of the project, political candidates are invited to answer a questionnaire designed to elicit the candidates political position on various issues. The project currently operates a Web site where users are also invited to fill out a questionnaire. Based upon the users answers, Smartvote determines which candidate most closely matches the views the user has expressed.

For our project, we investigated the answers of candidates of the national elections 2003 and 2007 of the five largest Swiss parties: SVP (national-conservative), SPS (social democrats), FDP (liberals), CVP (christian democrats) and GPS (Green party). Return was higher in 2007 (between 82-98% of all candidates answered the questionnaire) than 2003 (46-78%). The questionnaire consisted of 70 (2003) resp. 73 (2007) questions with multiple-choice answers (strongly agree, agree, disagree, strongly disagree). The distance between two candidates X and Y of the same party is calculated as follows: the possible answers are coded with 1 to 4. If both agreed to a question, the resulting value is 0, whereas the maximal value is 3 (strongly agree vs. strongly disagree, $|1-4|$). The sum of the absolute values for each question normalized with the number of answered questions is the distance between X and Y. The mutual comparison between all candidates of a party in 2003 or 2007 results in a distance matrix.

B. Coherence of political parties in Switzerland

Applying our measure of both $C_{dynamic}$ and C_{static} reveals the following result (see Fig. 4):

- FDP has a large dynamic coherence and low diversity. This is in some disagreement with the self-perception of the party.
- CVP is the less stablest and most diverse party, which is in some agreement with the general perception of this party.
- SPS is less stable and quite diverse and has lost both stability and diversity in 2007 compared to 2003.
- SVP was quite diverse in 2003, as SSC revealed two large blocks that disappeared in 2007.
- GPS was highly diverse in 2003 but consisted of several stable sub-clusters – i.e. was the only party located in the 'schisma-zone'.

This analysis demonstrates the strengths and weakness of a quantitative approach towards coherence: The method reveals that the GPS was in high danger of splitting in 2003 – which indeed was the case in

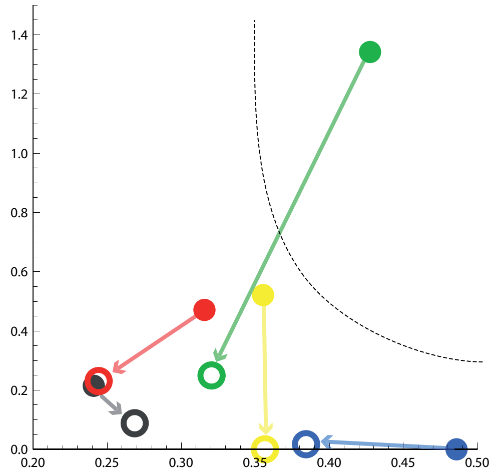

Fig. 4. The coherence of the largest Swiss parties in 2003 (filled dot) and 2007. Yellow: SVP, red: SPS, blue: FDP, black: CVP, green: GPS.

2004, when the Green Liberals were formed. In 2007, the GPS gained coherence doe to this split. However, the method was not able to predict the split of the SVP after 2007, although one has to add that a much smaller fraction of party members was affected by this split compared to the green voters potential.

V. Conclusion and outlook

We are aware that this analysis requires further investigations. First, differences in methodology between the questionnaires in 2003 and 2007, as well as the larger response rate in 2007 should be considered. Furthermore, we investigate alternatives to calculate $C_{dynamic}$ and C_{static}. But we belief that this novel approach for defining and quantifying the coherence of belief systems helps social science and humanities to understand important aspects of social change and its underlying beliefs and values.

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